An Improved Unimodal Absorbing Boundary Condition for Waveguide Problems

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***Abstract*—An improved unimodal absorbing boundary condition (ABC) is proposed by using one-dimensional (1-D) mode finite-difference time-domain (FDTD). In the unimodal ABC, an uniform auxiliary waveguide of input–output port should be introduced so that evanescent waves attenuate sufficiently. In this letter, the uniform auxiliary waveguide is simulated by 1-D mode FDTD rather than three-dimensional (3-D) FDTD which is used in the conventional unimodal ABC. Memory storage and CPU time are significantly reduced by applying the proposed ABC. A WG-90 rectangular waveguide with a thick asymmetric iris is analyzed by FDTD with the conventional modal ABC and the proposed ABC, and scattering parameters and computational efficiency are compared.**

***Index Terms*—Absorbing boundary condition, FDTD methods, waveguide.**

I. Introduction

Since modal absorbing boundary condition (ABC) has a very good wide-band absorbing characteristics, it is an appropriate ABC for waveguides with high dispersion. Modal ABC can be divided into two groups—unimodal ABC [1]–[3] and multimodal ABC [4], according to the number of modes absorbed. The dominant mode is taken into account in the unimodal ABC, while evanescent modes as well as the dominant mode must be considered in the multimodal ABC. Since it is complex to apply the multimodal ABC, the unimodal ABC is preferred and widely used for waveguide problems [1]–[3], [5].

A major disadvantage of the unimodal ABC is that the absorbing wall must be placed far enough from scattering objects to attenuate evanescent modes sufficiently. Therefore, considerable numerical costs are required when one simulates the uniform auxiliary waveguide between the absorbing wall and scattering objects by three-dimensional (3-D) finite-difference time-domain (FDTD) as in the conventional unimodal ABC. To overcome the disadvantage, we propose an improved unimodal ABC using one-dimensional (1-D) mode FDTD in the uniform auxiliary waveguide. Recently, similar ideas have been also developed independently in [6].

II. Formulation

We consider the structure of the waveguide with a discontinuity. In the conventional unimodal ABC, the entire structure including the uniform auxiliary waveguide is simulated by 3-D FDTD, which require a long CPU time and a considerable amount of memory storage [see Fig. 1(a)]. To improve the computational efficiency, a novel algorithm is proposed. The proposed algorithm is based on applying 1-D mode FDTD for the uniform auxiliary waveguide of the input–output port, as shown in Fig. 1(b). Simulating the uniform auxiliary waveguide in 1-D rather than 3-D leads to a significant improvement of numerical costs.

Since the uniform waveguide region of the input–output port can be analyzed in separate modes, one can apply an appropriate ABC for each mode, respectively. We propose that modal ABC be applied for the dominant mode and short-circuit condition be used for evanescent modes (the evanescent modal amplitudes are set to zero). Interfacing 3-D meshes and 1-D nodes is needed in the proposed algorithm. Conversion of 3-D formulation to 1-D formulation is done by mode expansion on the mode expansion plane, while conversion of 1-D formulation to 3-D formulation is done by mode composition on the mode composition plane [5]. Mode expansion/composition planes are depicted in Fig. 1(b). For a z-directed waveguide, mode composition and mode expansion can be presented as

$$\vec{E}_t(x,y,z,t) = \sum_i V_i(z,t)\vec{e}_i(x,y)$$

$$V_i(z,t) = \frac{\iint_S \vec{E}_t(x,y,z,t)\cdot\vec{e}_i(x,y)\,dx\,dy}{\iint_S |\vec{e}_i(x,y)|^2\,dx\,dy}$$

where $\vec{E}_t$ is the transverse electric field, V_i is the modal amplitude of the order $i, \vec{e}_i$ is the transverse electric field pattern of the ith-order mode, and s is the waveguide cross section.

Since the regions where 1-D mode FDTD apply are uniform, no coupling between modes takes place. Therefore, each modal amplitude V_i can be updated by the following difference equation [5]:

$$V_i^{n+1}(k) = \frac{c^2\Delta t^2}{\Delta z^2}[V_i^n(k+1) - 2V_i^n(k) + V_i^n(k-1)] + (2 - c^2\Delta t^2 k_{ci}^2)V_i^n(k) - V_i^{n-1}(k)$$

where $c, k_{ci}, n,$ and k are the velocity of the light in the waveguide, cutoff wavenumber of the ith-order mode, the time step, and the grid position along z axis, respectively.

Manuscript received June 3, 1997. This work was supported by the Korean Science and Engineering Foundation (KOSEF) under Grant 961-0922-120-2.

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Publisher Item Identifier S 1051-8207(97)08183-X.

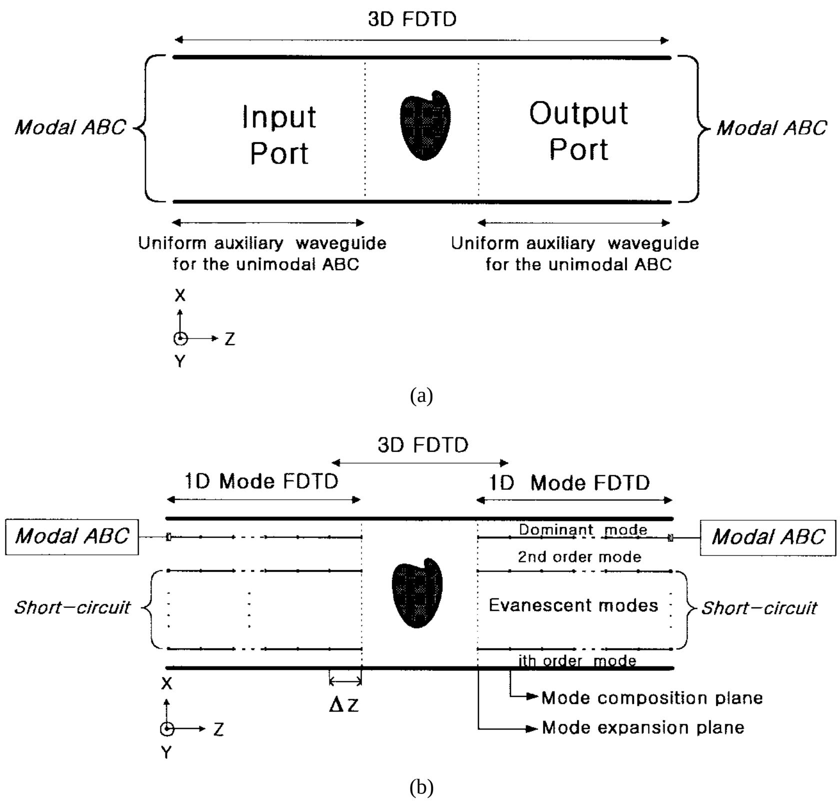

Fig. 1. (a) The conventional unimodal ABC. The uniform auxiliary waveguide of input–output port is simulated by 3-D FDTD. (b) The proposed unimodal ABC. The uniform auxiliary waveguide of input–output port is simulated by 1-D mode FDTD.

To terminate the 1-D computational domain, we use 1-D modal ABC for the dominant mode and apply the short-circuit condition for the evanescent modes. For the dominant mode, the modal amplitude on the absorbing wall is obtained by convolving the discrete Green's function with the modal amplitude on the plane Δz away from the absorbing wall:

$$V_1(k\text{abc}\Delta\text{z}, \text{n}\Delta\text{t}) = \sum_{m=0}^{n} G_1(m\Delta t)V_1((k\text{abc}-1)\Delta\text{z}, (\text{n}-\text{m})\Delta\text{t})$$

where G_1, n, and $k\text{abc}$ are the discrete Green's function for the dominant mode, the time step, and the z-directed grid position at the absorbing wall, respectively. G_1 can be obtained by pre-simulating the semi-infinite waveguide with the technique of recursive modeling [1].

III. Numerical Example

This procedure has been applied to the analysis of a WG-90 rectangular waveguide with a thick asymmetric iris as shown in Fig. 2. The space steps used are $\Delta x = 0.381$ mm, $\Delta y = 1.016$ mm, and $\Delta z = 0.47625$ mm, and the number of time iterations is 6000. The excitation pulse is a Gaussian pulse modulated by a sinusoid, with a frequency range from 8.2 to 12.4 GHz. Ten modes are adopted in both the proposed algorithm and the multimodal ABC. For the conventional unimodal ABC, the length of the 3-D uniform auxiliary waveguide is chosen as 45.72 mm (96 Δz), which is double the cutoff wavelength of the TE_{20} mode [1]. The length of the 3-D uniform auxiliary waveguide and that of the 1-D uniform auxiliary waveguide for the proposed unimodal ABC are set to $18\Delta z$ and $78\Delta z$, respectively. The length of the 3-D uniform auxiliary waveguide for the multimodal ABC is chosen as $18\Delta z$. Fig. 3 shows the scattering parameters obtained for the given structure. All FDTD simulations with three types of ABC's agree very well with each other and also agree well with mode-matching method. Table I shows that the proposed unimodal ABC has an improved computational efficiency than that of the conventional modal ABC. It is noted that an additional computational efficiency can be obtained by adjusting 1-D uniform auxiliary waveguide of each evanescent mode according to its attenuation characteristics.

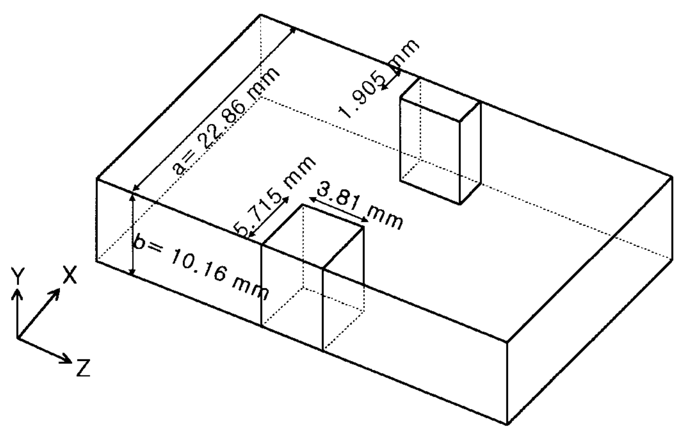

Fig. 2. A WG-90 rectangular waveguide with a thick asymmetric iris.

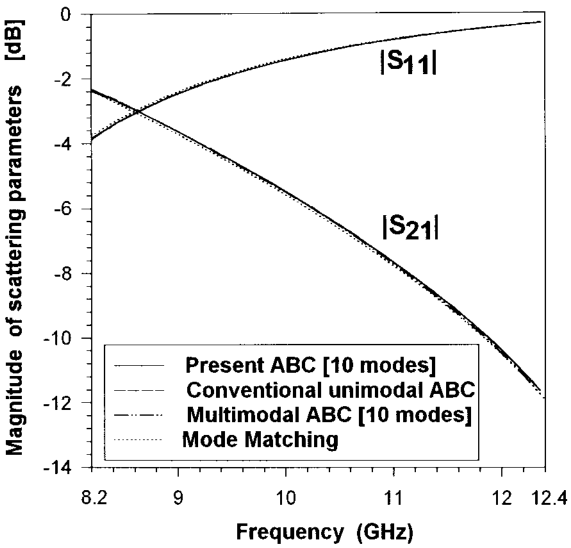

Fig. 3. Scattering parameters of rectangular waveguide with a thick asymmetric iris.

IV. Conclusion

An improved unimodal ABC is proposed by using 1-D mode FDTD. A WG-90 rectangular waveguide with a thick asymmetric iris is analyzed by FDTD with the conventional modal ABC and the novel unimodal ABC. The results show that the proposed algorithm provides improved computational efficiency without any considerable degradation of the accuracy.

TABLE I
Comparison of CPU Time and Memory Storage

Type of ABC	Total CPU time	Normalized memory storage
Conventional unimodal ABC	**6863 sec.**	**1**
Multimodal ABC [10 modes]	**2650 sec.**	**0.292**
Present ABC [10 modes]	**1894 sec.**	**0.236**

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